

-: Fluid Dynamics :-Sarjita SahooIntroduction :-

Fluid dynamics includes the study of forces causing fluid flow. Thus dynamics of fluid flow is the study of fluid motion with the force causing flow.

Equations of motion :-

In the fluid, the following forces are present.

- (i) f_g , gravity force
- (ii) f_p , pressure force
- (iii) f_v , force due to viscosity
- (iv) f_t , force due to turbulence
- (v) f_c , force due to compressibility

The net force,

$$F_x = (f_g)_x + (f_p)_x + (f_v)_x + (f_t)_x + (f_c)_x$$

- (i) If f_c is negligible,

$$F_x = (f_g)_x + (f_p)_x + (f_v)_x + (f_t)_x \text{ and}$$

this equation of motions are called

Reynold's equations of motion.

- (ii) For flow, whence f_t is negligible, the resulting equations of motion are known as Navier-Stokes equation or N-S equation.

- (iii) If the flow is assumed to be ideal, viscous force (f_v) is zero, and equation of motion are known as Euler's equation of motion.

Euler's equation of motion

- ⇒ This eqⁿ of motion in which the forces due to gravity and pressure are taken into consideration.
- ⇒ This is derived by considering the motion of a fluid element along a stream line.
- ⇒ Consider a stream line in which flow is taking place in s-direction.

Cross section = dA

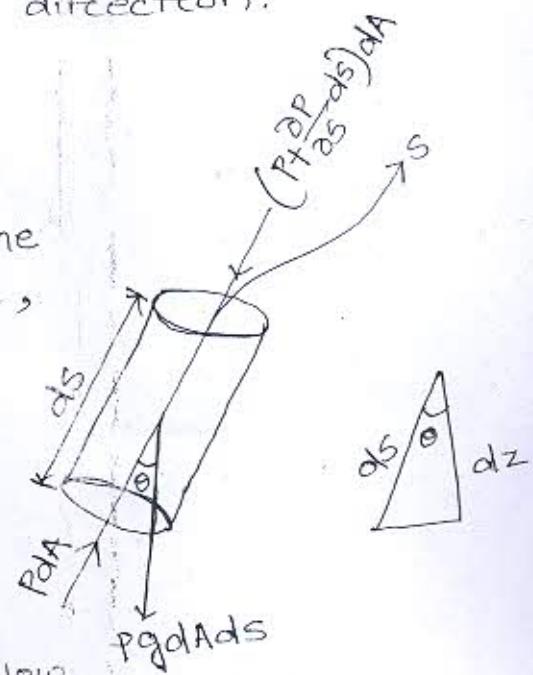
Length = ds

The forces acting on the cylindrical elements are,

(i) Pressure force PdA in the direction of flow.

(ii) Pressure force $(P + \frac{\partial P}{\partial s} ds)dA$ opposite to the direction of flow.

(iii) Weight of element $\rho g dA ds$.



Let, θ = Angle between the direction of flow and line of action of the wt. of element

The resultant force on the fluid element in the direction of s must be equal to the mass of fluid \times acceleration in the direⁿ s.

$$PdA - (P + \frac{\partial P}{\partial s} ds)dA = -\rho g dA ds \cos\theta = -\rho dA ds \times a_s$$

a_s is the acceleration in s direction.

$$a_s = \frac{dv}{dt}$$

$$= \frac{\partial v}{\partial s} \cdot \frac{ds}{dt} + \frac{\partial v}{\partial t}$$

$$= v \frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}$$

If the flow is steady, $\frac{\partial V}{\partial t} = 0$

$$a_s = \frac{V \partial V}{\partial s}$$

Substituting the value of a_s in the eqⁿ,

$$-\frac{\partial P}{\partial s} ds dA - \rho g dA ds \cos\theta = \rho dA ds \times \frac{\partial V}{\partial s}$$

⇒ By dividing, $\rho ds dA$

$$\frac{-\partial P}{\rho \partial s} + g \cos\theta + V \frac{\partial V}{\partial s} = 0$$

We have $\cos\theta = \frac{dz}{ds}$

$$\frac{1}{\rho} \frac{dP}{ds} + g \frac{dz}{ds} + V \frac{dV}{ds} = 0$$

$$\Rightarrow \boxed{\frac{dP}{\rho} + g dz + V dV = 0}$$

This equation is known as Euler's eqⁿ of motion.

Bernoulli's equation from Euler's eqn / Energy eqn

Bernoulli's equation is obtained by integrating the Euler's equation of motion as,

$$\int \frac{dp}{\rho} + \int gdz + \int v dv = \text{constant}$$

If flow is incompressible, ρ is const.

$$\begin{aligned}\frac{P}{\rho} + gz + \frac{v^2}{2} &= \text{const.} \\ \Rightarrow \frac{P}{\rho g} + z + \frac{v^2}{2g} &= \text{const.} \\ \Rightarrow \left[\frac{P}{\rho g} + \frac{v^2}{2g} + z \right] &= \text{const.}\end{aligned}$$

Where,

$\frac{P}{\rho g}$ = Pressure energy per unit wt
or pressure head

$v^2/2g$ = Kinetic energy per unit wt
or kinetic head

z = Potential energy per unit wt.
or potential head.

Date → 20th Aug

Bernoulli's Equation for real fluid

The Bernoulli's equation was derived on the assumption that fluid is inviscid.

- But all real fluids are viscous and hence offer resistance to flow.
- Thus there are always some losses in fluid flows.

→ Thus the Bernoulli's equation for real fluids between points 1 and 2 is given as,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

Where h_L = Loss of energy between 1 and 2.

Assumptions of Bernoulli's equation

The following are the assumptions of Bernoulli's eqⁿ:

- i) The fluid is ideal i.e. viscosity is zero.
- ii) The flow is steady.
- iii) The flow is incompressible
- iv) The flow is irrotational
- v) A pipe through which water is flowing, is having diameters, 20 cm and 10 cm at the cross section 1 and 2 respectively. The velocity of water at section 1 is given 4 m/s Find the velocity head at sections 1 and 2 and also rate of discharge.

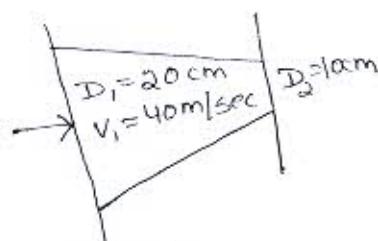
Given, $D_1 = 20 \text{ cm} = 0.2 \text{ m}$

$D_2 = 10 \text{ cm} = 0.1 \text{ m}$

$V_1 = 4 \text{ m/s}$

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times 0.2^2 = 0.0314 \text{ m}^2$$

$$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times 0.1^2 = 0.00785 \text{ m}^2$$



(i) Velocity head at section 1,

$$= \frac{V_1^2}{2g} = \frac{4 \times 4}{2 \times 9.81}$$

$$= 0.815 \text{ m}$$

(ii) Velocity head at section 2,

$$= \frac{V_2^2}{2g}$$

By applying continuity eqⁿ,

$$A_1 V_1 = A_2 V_2$$

$$\Rightarrow V_2 = \frac{A_1 V_1}{A_2}$$

$$= \frac{0.0314 \times 4}{0.00785}$$

$$= 16 \text{ m/s}$$

$$\text{Velocity head} = \frac{V_2^2}{2g}$$

$$= \frac{16^2}{2 \times 9.81}$$

$$= 83.047 \text{ m}$$

(iii) Rate of discharge = $A_1 V_1$ or $A_2 V_2$

$$= 0.0314 \times 4$$

$$= 125.6 \text{ litres/s.}$$



Q) Water is flowing through a pipe having diameters 300 mm and 200 mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 24.525 N/cm^2 and the pressure at the upper end is 9.81 N/cm^2 . Determine the difference in datum head if the ~~difference~~ the rate of flow through pipe is 40 lit/s.

Given,

$$D_1 = 300 \text{ mm} \\ = 0.3 \text{ m}$$

$$P_1 = 24.525 \text{ N/cm}^2 \\ = 24.525 \times 10^4 \text{ N/m}^2$$

$$D_2 = 200 \text{ mm} \\ = 0.2 \text{ m} \\ P_2 = 9.81 \text{ N/cm}^2 \\ = 9.81 \times 10^4 \text{ N/m}^2$$

Rate of flow = 40 lit/s
 $\Rightarrow Q = 0.04 \text{ m}^3/\text{s}$

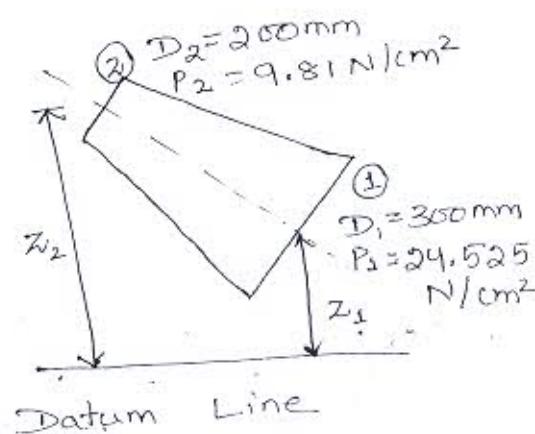
$$A_1 V_1 = A_2 V_2$$

$$V_1 = \frac{0.04}{A_1} = \frac{0.04}{\frac{\pi}{4} \times (0.3)^2} = 2.274 \text{ m/s} \quad 0.05658 \text{ m/s}$$

$$V_2 = \frac{0.04}{A_2} = \frac{0.04}{\frac{\pi}{4} \times (0.2)^2} = 1.247 \text{ m/s}$$

Applying Bernoulli's equation,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$



$$\Rightarrow \frac{24.525 \times 10^4}{1000 \times 9.81} + \frac{0.56^2}{2 \times 9.81} + z_1 \\ = \frac{9.81 \times 10^4}{1000 \times 9.81} + \frac{1.274^2}{2 \times 9.81} + z_2$$

$$\Rightarrow z_2 - z_1 = 13.697 \approx 13.70 \text{ m}$$

\therefore Difference in datum head $= z_2 - z_1 = 13.70 \text{ m.}$

- Q) Water is flowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm^2 and with mean velocity of 2 m/s. Find the total head or total energy per unit wt of the water at a c/s which is 5 m above the datum line.

\Rightarrow Given, $d = 0.5 \text{ m}$

$$P = 29.43 \text{ N/cm}^2 \\ = 29.43 \times 10^4 \text{ N/m}^2$$

$$V = 2 \text{ m/s}$$

$$z = 5 \text{ m}$$

$$\text{Total head} = \frac{P}{\rho g} + \frac{V^2}{2g} + z$$

$$\text{Pressure head} = \frac{P}{\rho g} = \frac{29.43 \times 10^4}{1000 \times 9.81} = 30 \text{ m}$$

$$\text{Kinetic head} = \frac{V^2}{2g} = \frac{2^2}{2 \times 9.81} = 0.204 \text{ m}$$

$$\text{Total head} = 30 + 0.204 + 5 \\ = 35.204 \text{ m.}$$

Date - 24th Aug

Practical applications of Bernoulli's equation
Bernoulli's equation is applied to some measuring devices;

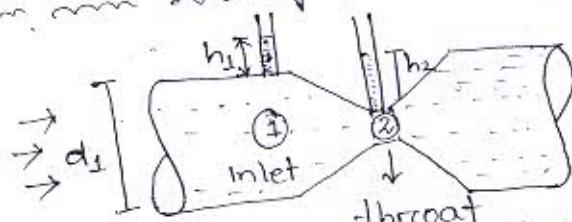
- 1) Venturimeter
- 2) Orifice meter
- 3) Pitot - tube

Venturimeter

A venturimeter is a device used for measuring the rate of flow of a fluid flowing through a pipe. It consists of three parts.

- (i) A short converging part
- (ii) Throat
- (iii) Diverging part

Expression for rate of flow through venturimeter



Consider a venturimeter fitted in a horizontal pipe, through which a fluid is flowing.

Let Let, d_1 = dia at inlet section

Ans
 P_1 = Pressure at section (1)

V_1 = Velocity at section (1)

a = area at section (1)

$$= \frac{\pi}{4} d_1^2$$

and d_2, P_2, V_2, a_2 are corresponding values at section 1 and 2, we get by applying

$$\text{Bernoulli's eq} \rightarrow \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

As pipe is horizontal,

$$\therefore z_1 = z_2$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\Rightarrow \frac{P_1 - P_2}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

By substituting the pressure head difference

$$\frac{P_1 - P_2}{\rho g} = h$$

we get,

$$\Rightarrow h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

By applying continuity eqn,

$$a_1 V_1 = a_2 V_2$$

$$\Rightarrow V_2 = \frac{a_2 V_2}{a_1}$$

Q) What
a pre
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\Rightarrow Give

$$\Rightarrow h = \frac{V_2^2}{2g} - \frac{\left(\frac{a_2 V_2}{a_1}\right)^2}{2g}$$

$$= \frac{V_2^2}{2g} \left[1 - \frac{a_2^2}{a_1^2} \right]$$

$$= \frac{V_2^2}{2g} \left[\frac{a_1^2 - a_2^2}{a_1^2} \right]$$

Tot.

Press

$$\Rightarrow V_2^2 = 2gh \left[\frac{a_1^2}{a_1^2 - a_2^2} \right]$$

Kinet

$$\Rightarrow V_2 = \sqrt{2gh \left(\frac{a_1^2}{a_1^2 - a_2^2} \right)}$$

Total

$$\Rightarrow V_2 = \frac{a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

Discharge, $Q = a_2 V_2 = a_1 V_1$

$$= \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$$Q_{act} = C_d \times \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

Whence, $C_d = \text{co-efficient of venturimeter}$

$$\boxed{C_d < 1}$$

Value of h is given as by differential U-tube manometer,

Case-I $\Rightarrow h = x \left[\frac{s_h}{s_o} - 1 \right]$

s_h = sp. gr. of heavier liquid

s_o = sp. gr. of flowing fluid

x = difference of heavier liquid column
in U-tube.

Case-II \Rightarrow

$$h = x \left[1 - \frac{s_t}{s_o} \right]$$

s_t = sp. gr. of lighter liquid in U-tube

Case-III \Rightarrow

For inclined venturimeter,

$$h = \left(\frac{P_1}{\rho g} + z_1 \right) - \left(\frac{P_2}{\rho g} + z_2 \right) = x \left[\frac{s_h}{s_o} - 1 \right]$$

Q) A horizontal venturimeter with inlet and throat diameters 30 cm and 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet and the throat is 20 cm of mercury. Determine the rate of flow. Take $C_d = 0.98$

Given, inlet dia, $d_1 = 30\text{cm}$

$$a_1 = \frac{\pi}{4} d_1^2 \\ = \frac{\pi}{4} \times 30^2 \\ = 706.85 \text{ cm}^2$$

Dia of throat, $d_2 = 15\text{cm}$

$$a_2 = \frac{\pi}{4} \times 15^2 \\ = 176.7 \text{ cm}^2$$

$$C_d = 0.98$$

Reading of differential manometer
 $\alpha = 20\text{cm}$ of mercury

Difference of pressure head,

$$h = \alpha \left[\frac{S_h}{S_o} - 1 \right]$$

$$= 20 \left[\frac{13.6}{1} - 1 \right]$$

$$= 20 \times 12.6 = 252 \text{ cm of water}$$

The discharge through venturimeter,

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \sqrt{2 \times 9.81 \times 252}$$

$$= 125.756 \text{ lit/s}$$

Dt-26th Aug

- Q) The inlet and throat diameters of a horizontal venturimeter are 30 cm and 10 cm respectively. The liquid flowing through the meter is water. The pressure intensity at inlet is 13.734 N/cm² while the ~~vacuum (ve)~~ pressure head at the throat is 37 cm of mercury. Find the rate of flow. Assume that 4% of the differential head is lost between the inlet and throat. Find also the value of C_d for the venturimeter.

Given,

$$d_1 = 30 \text{ cm}$$

$$a_1 = \frac{\pi}{4} \times 30^2 = 706.85 \text{ cm}^2$$

$$d_2 = 10 \text{ cm}$$

$$a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$$

$$P_1 = 13.734 \text{ N/cm}^2 = 13.734 \times 10^4 \text{ N/m}^2$$

$$\text{Pressure head} = \frac{P_1}{\rho g} = \frac{13.734 \times 10^4}{1000 \times 9.81} = 14 \text{ m of water}$$

$$\frac{P_2}{\rho g} = -37 \text{ cm of mercury}$$

$$= \frac{-37 \times 13.6}{100} = -5.032 \text{ m of water.}$$

differential head,

$$h = \frac{P_1}{\rho g} - \frac{P_2}{\rho g}$$

$$= 14.0 + 5.032$$

$$= 19.032 \text{ cm}$$

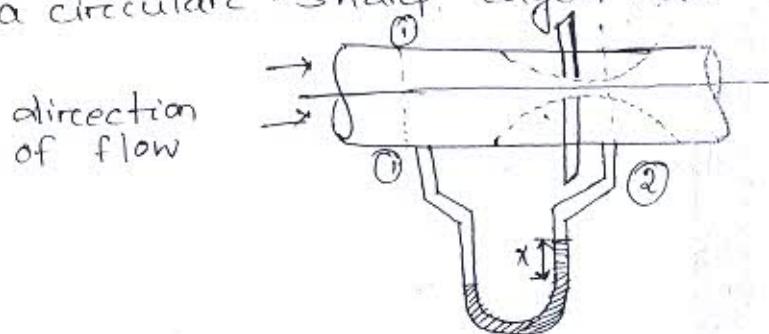
$$\text{Head lost} = 4\% \text{ of } h \\ = \frac{4}{100} \times 19.032 = 0.7613 \text{ m}$$

$$C_d = \sqrt{\frac{h-h_f}{h}} = \sqrt{\frac{19.032 - 0.7613}{19.032}} = 0.98$$

$$\text{discharge} \Rightarrow Q = C_d \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}} \\ = 0.98 \times \frac{706.85 \times 78.54 \sqrt{2 \times 9.81 \times 19.032}}{\sqrt{(706.85)^2 - (78.54)^2}} \\ = 0.14969 \text{ m}^3/\text{s}$$

Orifice Meter :-

- It is a device used for measuring the rate of flow of a fluid through a pipe.
- It is a cheaper device as compared to venturimeter.
- It also works on the same principle as that of venturimeter.
- It consists of a flat circular plate which has a circular sharp edged hole called orifice.



Let P_1 = Pressure at secⁿ 1

V_1 = Velocity at secⁿ 1

a_1 = area of pipe at secⁿ 1

and P_2 , V_2 and a_2 are corresponding value
at section 2.

Applying Bernoulli's eq? at secⁿ (1) and (2)

we get,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\Rightarrow \left(\frac{P_1}{\rho g} + z_1 \right) - \left(\frac{P_2}{\rho g} + z_2 \right) = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$\Rightarrow \text{But, } \left(\frac{P_1}{\rho g} + z_1 \right) - \left(\frac{P_2}{\rho g} + z_2 \right) = h = \text{differential head}$$

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$\Rightarrow 2gh = V_2^2 - V_1^2$$

$$\Rightarrow V_2 = \sqrt{2gh + V_1^2} \quad \text{--- (i)}$$

Where, C_c = co-efficient of contraction

$$a_2 = a_0 \times C_c$$

By continuity eqn,

$$a_1 V_1 = a_2 V_2$$

$$\Rightarrow V_1 = \frac{a_2}{a_1} V_2$$

$$= \frac{a_0 C_c}{a_1} V_2$$

Substituting the value of V_1 in equation (i) we get,

$$V_2 = \sqrt{2gh + \frac{a_0^2 C_c^2 V_2^2}{a_1^2}}$$

$$\Rightarrow V_2^2 = 2gh + \left(\frac{a_0}{a_1}\right)^2 C_c^2 V_2^2$$

$$\Rightarrow V_2^2 \left[1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2 \right] = 2gh$$

$$\Rightarrow V_2^2 = \frac{2gh}{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}$$

The discharge,

$$Q = V_2 \times a_2$$

$$= V_2 \times a_0 C_c$$

$$= \frac{a_0 C_c \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$$C_d = C_c \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$$Q = a_0 \times C_d \times \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$
$$= \frac{C_d a_0 \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}} = \boxed{\frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}} = Q}$$

Q8) An orifice meter with orifice diameter 10cm is inserted in a pipe of 20cm diameter. The pressure gauges fitted upstream and downstream of the orifice meter gives readings of 19.62 N/cm^2 and 9.81 N/cm^2 respectively. Co-eff. of discharge for the orifice meter is given as 0.6. Find the discharge of water through pipe.

Given,

$$\text{Dia of orifice, } d_0 = 10 \text{ cm}$$

$$\text{Area, } a_0 = \frac{\pi}{4} \times (10)^2 \\ = 78.54 \text{ cm}^2$$

$$\text{Dia of pipe, } d_1 = 20 \text{ cm}$$

$$a_1 = \frac{\pi}{4} \times (20)^2 \\ = 314.16 \text{ cm}^2$$

$$P_1 = 19.62 \text{ N/cm}^2 \\ = 19.62 \times 10^4 \text{ N/m}^2$$

$$\frac{P_1}{\rho g} = \frac{19.62 \times 10^4}{1000 \times 9.81} = 20 \text{ m of water}$$

$$\frac{P_2}{\rho g} = \frac{9.81 \times 10^4}{1000 \times 9.81} = 10 \text{ m of water}$$

$$h = \frac{P_1}{\rho g} - \frac{P_2}{\rho g} = 20 - 10 = 10 \text{ m of water}$$

$$C_d = 0.6$$

$$\dot{Q} = C_d \frac{a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

$$= 0.6 \times \frac{314.16 \times 78.54 \sqrt{2 \times 9.81 \times 10}}{\sqrt{314.16^2 - 78.54^2}}$$

$$= 68213.28 \text{ cm}^3/\text{s}$$

$$= 68.21 \text{ lit/s.}$$

Q) An orifice meter with orifice diameter 15 cm is inserted in a pipe of 30 cm dia. The pressure difference measured by a mercury oil differential manometer on the two sides of the orifice meter gives a reading of 50 cm of mercury. Find the rate of flow of oil sp. gr. 0.9 when the coeff of discharge of the orifice meter = 0.64

Given,

$$d_0 = 15 \text{ cm}$$

$$a_0 = \frac{\pi}{4} (15)^2 = 176.7 \text{ cm}^2$$

$$d_1 = 30 \text{ cm}$$

$$a_1 = \frac{\pi}{4} \times 30^2 = 706.85 \text{ cm}^2$$

$$\text{sp. gr. of oil} \Rightarrow S_o = 0.9$$

Reading of diff. manometer $\Rightarrow x = 50 \text{ cm}$ of mercury

$$\text{difference head, } h = x \left[\frac{S_o}{S_o - 1} \right]$$

$$= 50 \left[\frac{13.6}{1} - 1 \right]$$

$$= 705.5 \text{ cm of oil}$$

The rate of flow,

$$Q = C_d \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \sqrt{2gh}$$

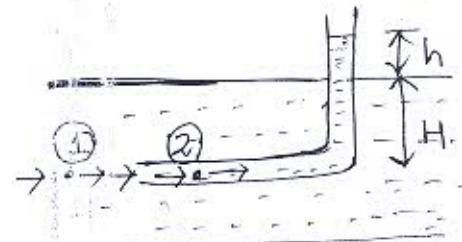
$$= 0.64 \times \frac{176.7 \times 706.85}{\sqrt{706.85^2 - 176.7^2}} \times \sqrt{2 \times 981 \times 705.5}$$

$$= 137414.25 \text{ cm}^3/\text{s}$$

$$= 137.414 \text{ lit/s.}$$

Pitot Tube

It is a device used for measuring the velocity of flow at any point in a pipe or a channel. It is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of KE into pressure energy.



→ The velocity is determined by measuring the rise of liquid in the tube.

Consider two points (1) and (2) at the same level in such a way that point (2) is just at the inlet of the pitot tube and point (1) is far away from the tube.

Let, P_1 = intensity of pressure at point 1

V_1 = velocity of flow at (1)

P_2 = Pressure at point (2)

V_2 = Velocity at point (2) which is 0

z_1 = depth of tube in the liquid

h = rise of liquid in the tube above free surface.

By applying Bernoulli's eqⁿ in (1) & (2) sec?

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\text{But, } z_1 = z_2 \quad V_2 = 0$$

$$\text{and } \frac{P_1}{\rho g} - \frac{P_2}{\rho g} = h$$

$$\frac{P_1}{\rho g} = \text{Pressure head at } (1) = H$$

$$\frac{P_2}{\rho g} = \text{Pressure head at } (2) = h + H$$

Substituting this value we get,

$$H + \frac{V_1^2}{2g} = h + H$$

$$\Rightarrow h = \frac{V_1^2}{2g}$$

$$\Rightarrow V_1 = \sqrt{2gh}$$

$$\text{Actual velocity, } (V_1)_{\text{act}} = C_v \sqrt{2gh}$$

C_v = co-efficient of pitot tube

If the difference of the levels of the manometers liquid say α . Then $h = \alpha \left[\frac{\rho g}{30} - 1 \right]$

- Q) A pitot-static tube placed on the centre of a 300mm pipe line has one orifice pointing upstream and other perpendicular to it. The mean velocity in the pipe is 0.80 of the central velocity. Find the discharge through the pipe if the pressure difference bet" the two orifices is 60 mm of water. Take $C_v = 0.98$.

Given,

$$\text{dia of pipe} \Rightarrow d = 300 \text{ mm} = 0.30 \text{ m}$$

$$\text{pressure head} \Rightarrow h = 60 \text{ mm of water} \\ = 0.06 \text{ m of water}$$

$$C_v = 0.98$$

$$\text{mean velocity, } \bar{V} = 0.80 \times \text{central velocity}$$

$$\text{Central Velocity} = C_v \sqrt{2gh}$$

$$= 0.98 \times \sqrt{2 \times 9.81 \times 0.06} \\ = 1.063 \text{ m/s}$$

$$\bar{V} = 0.80 \times 1.063 = 0.8504 \text{ m/s}$$

$$\text{Discharge} = \text{Area of pipe} \times \bar{V}$$

$$= \frac{\pi}{4} d^2 \times \bar{V}$$

$$= \frac{\pi}{4} \times 0.30^2 \times 0.8504$$

$$= 0.06 \text{ m}^3/\text{s}$$

- Flow Through Pipes :-

Dt-29th Aug

When Reynolds no. is less than 2000 for pipe flow, the flow is known as laminar flow whereas when the Reynolds no is more than 4000, the flow is known as turbulent flow.

$$\text{Reynold's No.} = \text{Re} = \frac{\rho V D}{\mu} = \frac{V D}{\eta}$$

$\text{Re} < 2000$, laminar

$\text{Re} > 4000$, turbulent

$2000 < \text{Re} < 4000$, transition

→ The turbulent flow of fluids through pipes running full will be considered, and in this condition fluid is flowing under high pressure.
Ex:- Water supply lines.

→ But, in case of partially filled cross section fluid is flowing under gravitational force and the flow type is laminar.

Ex:- Sewer lines

Loss of energy in pipes

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost.

The loss of energy is classified as;

Energy Losses

↓
Major energy loss

This is due to friction and it is calculated by the following formula

- (a) Darcy weisbach
- (b) Chezy's formula

↓
Minor energy loss

- This is due to
- (a) Sudden expansion
 - (b) Sudden contraction
 - (c) Bend in pipe
 - (d) Pipe fitting
 - (e) An obstruction

Loss of energy due to friction

(a) Darcy - Weisbach formula :-

The loss of head (or energy) in pipes due to friction is calculated from Darcy - Weisbach equation which has been given by ,

$$h_f = \frac{4 f L V^2}{2 g d}$$

Where, h_f = loss of head due to friction

f = Co-efficient of friction

$$= \frac{16}{Re} \text{ for } Re < 2000$$

$$= \frac{0.079}{Re^{1/4}} \text{ for } Re > 4000$$

L = length of pipe

V = mean velocity

d = dia of pipe

(b) Chezy's formula :-

Chezy's formula for loss of head due to friction in pipe, is given by ,

$$h_f = \frac{f'}{4g} \times \frac{P}{A} \times L \times V^2$$

Where, h_f = Loss of head due to friction

A = area of c/s

V = mean velocity

P = Wetted perimeter

L = length of pipe.

The ratio of $\frac{A}{P}$ is called hydraulic mean depth.

Which is denoted by m .

$$\begin{aligned}\text{Hydraulic mean depth, } m &= \frac{A}{P} \\ &= \frac{\pi/4 d^2}{\pi d} \\ &= \frac{d}{4}\end{aligned}$$

$$\text{Substituting, } \frac{P}{A} = \frac{1}{m} = \frac{4}{d}$$

$$h_f = \frac{f'}{fg} \times L \times v^2 \times \frac{1}{m}$$

$$v^2 = h_f \times \frac{fg}{f'} \times m \times \frac{1}{L}$$

$$v = \sqrt{\frac{fg}{f'} \times m \times \frac{h_f}{L}}$$

Let, $\sqrt{\frac{fg}{f'}} = C$, where C is a constant known as chezy's constant.

$$\frac{h_f}{L} = i, \text{ where } i \text{ is loss of head per unit length}$$

Substituting these values,

$$v = C \sqrt{m i}$$

- Q) Find the head lost due to friction in a pipe of diameter 300 mm and length 50 m through which water is flowing at a velocity of 3 m/s using
 (i) Darcy formula
 (ii) Chezy's formula for which $C = 60$
 Take ν for water = 0.01 Stoke

Given,

$$\text{dia of pipe} = d = 300 \text{ mm} = 0.30 \text{ m}$$

$$\text{length of pipe} = L = 50 \text{ m}$$

$$\text{velocity of flow} = V = 3 \text{ m/s}$$

$$\text{Chezy's const} = C = 60$$

$$\begin{aligned}\text{Kinematic viscosity, } \nu &= 0.01 \text{ Stoke} \\ &= 0.01 \text{ cm}^2/\text{s} \\ &= 0.01 \times 10^{-4} \text{ m}^2/\text{s}\end{aligned}$$

(i) Darcy formula is given by,

$$h_f = \frac{4fLV^2}{2gd}$$

$$Re = \frac{Vd}{\nu} = \frac{3 \times 0.3}{0.01 \times 10^{-4}} = 9 \times 10^5$$

$$f = \frac{0.079}{Re^{1/4}} = \frac{0.079}{(9 \times 10^5)^{1/4}} = 0.00256$$

$$\begin{aligned}\text{head lost, } h_f &= \frac{4 \times 0.00256 \times 50 \times 3^2}{2 \times 9.81 \times 0.3} \\ &= 0.7828 \text{ m}\end{aligned}$$

(ii) Chezy's formula,

$$V = C \sqrt{m i}$$

$$m = \frac{d}{4} = \frac{0.30}{4} = 0.075 \text{ m}$$

$$\Rightarrow 3 = 60 \sqrt{0.075 i}$$

$$\Rightarrow i = \frac{h_f}{L} = 0.333 \Rightarrow h_f = \frac{0.333 \times 50}{1.665} = 1.665 \text{ m}$$

Q) Cal. the discharge through a pipe of pipe 20 cm diameter when the difference of pressure head between the two ends of a pipe 500 m apart is 4m of water. Take the value of $f = 0.009$ in the formula $h_f = \frac{4fLV^2}{2gd}$

Given,

$$d = 200 \text{ mm} = 0.20 \text{ m}$$

$$L = 500 \text{ m}$$

$$h_f = 4 \text{ m of water}$$

$$f = 0.009$$

$$h_f = \frac{4fLV^2}{2gd}$$

$$\Rightarrow 4 = \frac{4 \times 0.009 \times 500 \times V^2}{0.2 \times 2 \times 9.81}$$

$$\Rightarrow V^2 = 0.872$$

$$\Rightarrow V = \sqrt{0.872}$$

$$= 0.934 \text{ m/s}$$

Discharge, $Q = \text{velocity} \times \text{Area}$
 $= 0.934 \times \frac{\pi}{4} d^2$
 $= 29.3 \text{ lit/s}$

31st Aug

X/ Find the dia of pipe of length 2000m when the rate of flow of water through the pipe is 200 lit/s and the head lost due to friction is 4m. Take, C=50 in chezy's formula.

Given,

$$L = 2000 \text{ m}$$

$$Q = 200 \text{ lit/s} \\ = 0.2 \text{ m}^3/\text{s}$$

$$h_f = 4 \text{ m}$$

$$C = 50$$

$$\text{Velocity } V = \frac{Q}{\text{Area}} = \frac{0.2}{\frac{\pi}{4} \times d^2} \\ = \frac{0.8}{\pi d^2}$$

Hydraulic mean depth,

$$m = d/4$$

$$i = \frac{h_f}{L} = \frac{4}{2000} = 0.002$$

Chezy's formula,

$$V = C \sqrt{m i}$$

$$\Rightarrow \frac{0.8}{\pi d^2} = 50 \sqrt{\frac{d}{4} \times 0.002}$$

$$\Rightarrow \frac{0.8}{\pi d^2} \times 50 = \cancel{0.00509}$$

$$\Rightarrow \left(\frac{0.8}{\pi d^2 \times 50} \right)^2 = \frac{d}{4} \times 0.002$$

$$\Rightarrow \frac{0.0000259}{d^4} = \frac{d}{4} \times 0.002$$

$$\Rightarrow d^5 = \frac{4 \times 0.0000259}{0.002}$$

$$\Rightarrow d = \sqrt[5]{0.0518}$$

$$= 0.553 \text{ m}$$

$$= 553 \text{ mm}$$

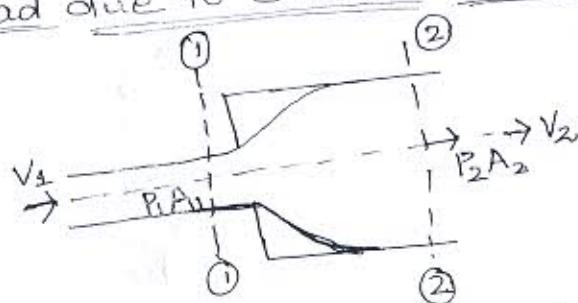
Minor Energy Losses

The loss of energy due to change of velocity of the flowing fluid is called minor loss of energy.

→ The minor loss of energy includes the following cases.

1. Loss of head due to sudden enlargement
2. Loss of head due to sudden contraction
3. Loss of head at the entrance of a pipe
4. Loss of head at the exit of pipe
5. Loss of head due to an obstruction in pipe.
6. Loss of head due to bend in pipe
7. Loss of head in various pipe fittings.

Loss of head due to sudden enlargement :-



Consider a liquid flowing through a pipe which has sudden enlargement.

Consider two sections (1)-(1) and (2)-(2) before and after the enlargement.

Let, P_1 = Pressure intensity at sec¹ 1-1

V_1 = Velocity of flow at sec¹ 1-1

A_1 = Area of pipe at sec¹ 1-1

$P_2, V_2 \text{ & } A_2$ are the corresponding values at sec¹ 2-2.

Due to sudden change in diameter of the pipe D_2 , the liquid flowing from the smaller pipe is not able to follow the abrupt change of the boundary.

- ↗ Thus the flow separates from the boundary and turbulent eddies are formed.
- ↗ The loss of head take place due to the formation of these eddies.

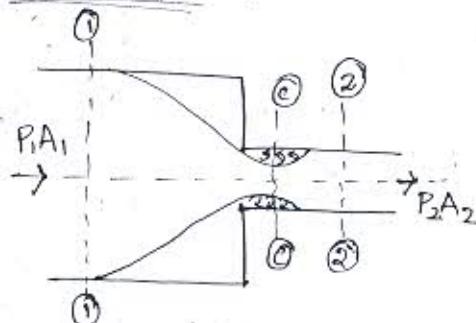
By applying Bernoulli's eqn in both sides the head loss due to sudden expansion is calculated by,

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

Loss of head due to sudden contraction

Dt - 2nd Sept

Consider a liquid flowing in a pipe which has a sudden contraction in area.



- ⇒ Consider two sections 1-1 and 2-2 before and after contraction.
- ⇒ As the liquid flows from large pipe to smaller pipe, the area of flow goes on decreasing and becomes minimum at section C-C.
- ⇒ The section C-C is called vena-contracta.
- ⇒ The loss of head due to sudden contraction is actually due to sudden enlargement from vena-contracta to smaller pipe.

Let,

A_c = Area of flow at section C-C

V_c = Velocity at section C-C

A_2 = Area of flow at section 2-2

V_2 = Velocity of flow at section 2-2

h_c = Loss of head due to sudden contraction

$$h_c = \frac{K V_2^2}{2g} = 0.375 \frac{V_2^2}{2g} \approx 0.5 \frac{V_2^2}{2g}$$

$$h_c = \frac{(V_c - V_2)^2}{2g} = \frac{V_2^2}{2g} \left[\frac{V_c}{V_2} - 1 \right]^2 = \frac{V_2^2}{2g} \left[\frac{1}{C_c} - 1 \right]^2$$

$$\text{Where, } K = \sqrt{\left[\frac{1}{C_c} - 1 \right]^2}$$

$$h_c = \frac{K V_2^2}{2g} = 0.375 \frac{V_2^2}{2g}$$

If the value of C_c is not given then consider,

$$h_c = 0.5 \frac{V_2^2}{2g}$$

Q) Find the loss of head when a pipe diameter 200mm is suddenly enlarged to a dia. of 400 mm. The rate of flow of water through the pipe is 250 lit/s.

Given,

$$D_1 = 200 \text{ mm} = 0.20 \text{ m}$$

$$D_2 = 400 \text{ mm} = 0.4 \text{ m}$$

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (0.2)^2 = 0.03141 \text{ m}^2$$

$$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times (0.4)^2 = 0.12564 \text{ m}^2$$

$$\text{Discharge, } Q = 250 \text{ lit/s} \\ = 0.25 \text{ m}^3/\text{s}$$

$$V_1 = \frac{Q}{A_1} = \frac{0.25}{0.03141} = 7.96 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.25}{0.12564} = 1.99 \text{ m/s}$$

$$h_E = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.96 - 1.99)^2}{2 \times 9.81} \\ = 1.816 \text{ m. of water.}$$

Q) At a sudden entrance

At a sudden enlargement of a water main from 240 mm to 480 mm diameter, the hydraulic gradient rises by 10 mm. Estimate the rate of flow.

Given,

$$D_1 = 240 \text{ mm} = 0.24 \text{ m}$$

$$D_2 = 480 \text{ mm} = 0.48 \text{ m}$$

$$A_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times 0.24^2$$

$$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times 0.48^2$$

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

By applying continuity eqⁿ,

$$A_1 V_1 = A_2 V_2$$

$$\Rightarrow V_1 = \frac{A_2 V_2}{A_1}$$

$$= \frac{\frac{\pi}{4} \times 0.48^2 \times V_2}{\frac{\pi}{4} \times 0.24^2}$$

$$= 4V_2$$

$$h_e = \frac{(4V_2 - V_2)^2}{2g} = \frac{9V_2^2}{2g}$$

Rise in hydraulic gradient = 10 mm

$$\Rightarrow \left(\frac{P_2}{\rho g} + z_2 \right) - \left(\frac{P_1}{\rho g} + z_1 \right) = 10 \text{ mm} = \frac{1}{100} \text{ m}$$

By applying Bernoulli's eqⁿ in both secⁿ 1 & 2,

We get,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_e$$

$$\frac{6.112}{2 \times 9.81} + h_e$$

$$\Rightarrow \frac{P_1}{\rho g} + \frac{(4V_2)^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_e$$

$$\Rightarrow \frac{16V_2^2}{2g} - \frac{V_2^2}{2g} + \frac{9V_2^2}{2g} = \left(\frac{P_2}{\rho g} + z_2 \right) - \left(\frac{P_1}{\rho g} + z_1 \right)$$

$$\Rightarrow \frac{16V_2^2 - V_2^2 + 9V_2^2}{2g} = \frac{1}{100}$$

$$\Rightarrow V_2^2 = \frac{2 \times 9.81}{600}$$

$$\Rightarrow V_2 = \sqrt{\frac{2 \times 9.81}{600}}$$

$$= 0.1808$$

$$\approx 0.181 \text{ m/s}$$

Discharge, $Q = A_2 V_2$

$$= \frac{\pi}{4} \times 0.48^2 \times 0.181$$

$$= 0.03275 \text{ m}^3/\text{s}$$

$$= 32.75 \text{ l/s}$$

Dt-13rd Sept.

- Q) A horizontal pipe of dia 500 mm is suddenly contracted to a diameter of 250 mm. The pressure intensities in the large and smaller pipe is given as 13.734 N/cm^2 and 11.772 N/cm^2 respectively. Find the loss of head due to contraction when rate of flow is 300 lit/s. Also find the value of C_c .

Given,

$$D_1 = 500 \text{ mm} = 0.5 \text{ m}$$

$$D_2 = 250 \text{ mm} = 0.25 \text{ m}$$

$$A_1 = \frac{\pi}{4} \times 0.5^2$$

$$A_2 = \frac{\pi}{4} \times 0.25^2$$

$$P_1 = 13.734 \times 10^4 \text{ N/m}^2$$

$$P_2 = 11.772 \times 10^4 \text{ N/m}^2$$

$$Q = 300 \text{ lit/s} = 0.3 \text{ m}^3/\text{s}$$

From continuity eqn,

$$A_1 V_1 = A_2 V_2$$

$$\Rightarrow V_1 = \frac{A_2 V_2}{A_1} = \frac{\frac{\pi}{4} \times 0.25^2 \times V_2}{\frac{\pi}{4} \times 0.5^2} = \frac{V_2}{4}$$

$$\Rightarrow V_2 = 4V_1$$

$$V_1 = \frac{Q}{A_1} = \frac{0.30}{\frac{\pi}{4} \times 0.5^2} = 1.528 \text{ m/s}$$

$$V_2 = 4 \times 1.528 = 6.112 \text{ m/s}$$

From Bernoulli's eqn,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_e$$

$$\text{Hence } Z_1 = Z_2$$

$$\Rightarrow \frac{13.74 \times 10^4}{1000 \times 9.81} + \frac{(1.528)^2}{2 \times 9.81} = \frac{11.772 \times 10^4}{1000 \times 9.81} + \frac{6.112^2}{2 \times 9.81} + h_e$$

$$\Rightarrow h_e = 0.215 \text{ m}$$

$$h_c = \frac{V_2^2}{2g} \left[\frac{1}{C_c} - 1 \right]^2$$

$$\Rightarrow 0.215 = \frac{6.112^2}{2 \times 9.81} \left[\frac{1}{C_c} - 1 \right]^2$$

$$\Rightarrow \left[\frac{1}{C_c} - 1 \right]^2 = \frac{0.215 \times 2 \times 9.81}{6.112^2}$$

$$= 0.1129$$

$$\Rightarrow \frac{1}{C_c} - 1 = \sqrt{0.1129}$$

$$\Rightarrow \frac{1}{C_c} = 0.336 + 1 = 1.336$$

$$\Rightarrow C_c = \frac{1}{1.336} = 0.748 \text{ m.}$$

Loss of head at the entrance of pipe

→ This is the loss of energy which occurs when a liquid enters a pipe which is connected to a large tank or reservoir.

→ For a sharp edge entrance, this loss is slightly more than a rounded or bell mouthed entrance.

→ This loss is denoted by h_i ,

$$h_i = 0.5 \frac{V^2}{2g}$$

Loss of head at the exit of pipe :-

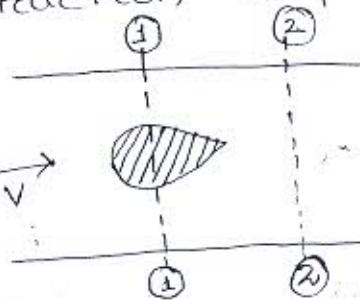
This loss of head due to the velocity of liquid at outlet of the pipe which is dissipated either in the form of a free jet or it is lost in the tank or reservoir.

This loss is denoted by h_o .

$$h_o = \frac{V^2}{2g}$$

Head Loss due to an obstruction in a pipe

Whenever there is an obstruction in a pipe, the loss of energy takes place due to reduction of the area of the cross section of the pipe at the place where obstruction is present.



Let,

a = Max. area of obstruction

A = area of pipe

V = Velocity of liquid in pipe.

Then $(A-a)$ = Area of flow of liquid at section 1-1

v_c = Velocity of liquid at vena-contracta

Head loss due to obstruction = loss of head due to enlargement from vena-contracta to sec' 2-2.

$$= \frac{(v_c - V)^2}{2g}$$

From continuity,

$$\text{We have } acv_c = AxV$$

Where ac = area of c/s at vena-contracta
 c_c = Co-eff of contraction.

$$\text{Then, } c_c = \frac{\text{area at vena-contracta}}{(A-a)}$$

$$= \frac{ac}{A-a}$$

$$\Rightarrow a_c = C_c \times (A - a)$$

Substituting this value,

$$C_c \times (A - a) V_c = A \times V$$

$$\Rightarrow V_c = \frac{A \times V}{C_c (A - a)}$$

Head loss due to obstruction

$$= \frac{(V_c - V_a)^2}{2g}$$

$$= \frac{\left(\frac{A \times V}{C_c (A - a)} - V \right)^2}{2g}$$

$$\boxed{\text{Head loss due to obst} = \frac{V^2}{2g} \left(\frac{A}{C_c (A - a)} - 1 \right)^2}$$

Head loss due to bend in pipe :-

When there is any bend in a pipe, the velocity of flow changes due to which the separation of the flow from the boundary and also formation of eddies takes place. Thus the energy is lost.

Loss of head due to bend is expressed as,

$$\boxed{h_b = \frac{K V^2}{2g}}$$

The value of K depends upon,

- (i) Angle of bend
- (ii) Radius of curvature of bend
- (iii) Diameter of pipe.

Loss of head in various pipe fittings :-

The loss of head in the various pipe fittings such as valves, coupling etc. is expressed as,

$$= \frac{K V^2}{2g}$$

Where, V = Velocity of flow

K = Co-eff. of pipe fitting

- Q) Water is flowing through a horizontal pipe of diameter 200 mm at a velocity of 3 m/s. A circular solid plate of diameter 150 mm is placed in the pipe to obstruct the flow. Find the loss of head due to obstruction in the pipe if $C_c = 0.62$.

Given,

$$D = 200 \text{ mm} = 0.2 \text{ m}$$

$$V = 3 \text{ m/s}$$

$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times 0.2^2 = 0.03141 \text{ m}^2$$

dia of obstruction, $d = 150 \text{ mm} = 0.15 \text{ m}$

$$\text{area of } " \quad , \quad a = \frac{\pi}{4} \times 0.15^2 \\ = 0.01767 \text{ m}^2$$

$$C_c = 0.62$$

Head lost due to obstruction, z

$$= \frac{V^2}{2g} \left(\frac{A}{C_c(A-a)} - 1 \right)^2$$

$$= \frac{3^2}{2 \times 9.81} \left(\frac{0.03141}{0.62(0.03141 - 0.01767)} - 1 \right)^2$$

$$= 3.311 \text{ m}$$

Q) A pipe line AB of dia 300 mm and of length 400 m carries water at the rate of 50 lit/s. The flow takes place from A to B where point B is 30 m above A. Find the pressure at A if the pressure at B is 19.62 N/cm². Take f = 0.008.

Given,

$$d = 300 \text{ mm} = 0.3 \text{ m}$$

$$L = 400 \text{ m}$$

$$Q = 50 \text{ lit/s} = 0.05 \text{ m}^3/\text{s}$$

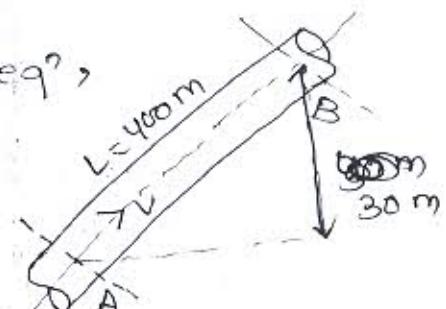
$$V = \frac{Q}{\text{Area}} = \frac{0.05}{\frac{\pi}{4} d^2} = \frac{0.05}{\frac{\pi}{4} \times 0.3^2} = 0.7074 \text{ m/s}$$

Pressure at B,

$$P_B = 19.62 \text{ N/cm}^2 \\ = 19.62 \text{ N/m}^2$$

$$f = 0.008$$

Applying Bernoulli's eqⁿ,



$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + z_B + h_f$$

$$\Rightarrow \text{But, } V_A = V_B \\ z_A = 0, z_B = 30$$

$$h_f = \frac{4f L V^2}{2g d}$$

$$(i) \quad \frac{P_A}{\rho g} + 0 + 0 = \frac{19.26 \times 10^4}{1000 \times 9.81} + 30 + \frac{4 \times 0.008 \times 400 \times 0.7074^2}{2 \times 9.81 \times 0.3}$$

$$\Rightarrow P_A = 51.088 \times 1000 \times 9.81$$

$$\Rightarrow P_A = 50.12 \text{ N/cm}^2$$

Hydraulic gradient and Total energy line :-

- Hydraulic gradient line is defined as the line which gives the sum of pressure head ($\frac{P}{\rho g}$) and datum head (z) of a flowing fluid in a pipe with respect to some reference line.
- HGL is the line which is obtained by joining the top of all vertical coordinates showing the pressure head ($\frac{P}{\rho g}$) of a flowing fluid in a pipe from the centre of pipe.

Total Energy Line :-

- It is defined as the line which gives the sum of pressure head, datum head and Kinetic head of a flowing fluid in a pipe with respect to some reference line.
- It is also defined as the line which is obtained by joining all vertical coordinates showing the sum of pressure head and kinetic head from the centre of the pipe.
 - It is written as T.E.L

Q) Water is flowing through a dia 20cm and length 50m when one end of the pipe is connected to a tank and other end of the pipe is open to the atmosphere. The pipe is horizontal and the ht. of water in tank is 4m above the centre of the pipe. consider all minor loss and tank and take $f = 0.009$

$$h_f = \frac{4fLV^2}{2gd}, V = 2.734 \text{ m/s. Draw HGL \times TEL}$$

$$\Rightarrow \text{Given, } L = 50\text{m}$$

$$d = 200\text{mm} = 0.2\text{m}$$

$$H = 4\text{m}$$

$$f = 0.009$$

$$V = 2.734 \text{ m/s.}$$

h_i = head lost at entrance of pipe

$$= 0.5 \frac{V^2}{2g}$$

$$= 0.5 \times \frac{2.734^2}{2 \times 9.81}$$

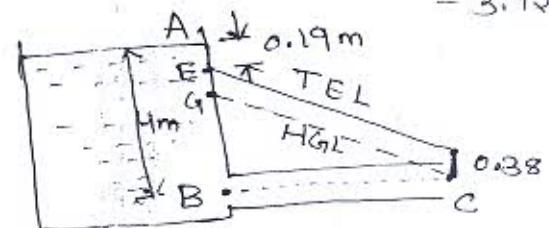
$$= 0.19\text{m.}$$

h_f = head loss due to friction

$$= \frac{4fLV^2}{2gd} = \frac{4 \times 0.009 \times 50 \times (2.734)^2}{2 \times 9.81 \times 0.2}$$

$$= 3.498$$

(a) Total energy line :-



$$\text{Total energy at A} = \frac{P}{\rho g} + \frac{V^2}{2g} + z$$

$$= 0 + 0 + 4 = 4\text{m}$$

$$\text{Total energy at B} = \text{Total energy at A} - h_i$$

$$= 4 - 0.19 = 3.81\text{m}$$

$$\text{Total energy at C} = \frac{P_c}{\rho g} + \frac{V_c^2}{2g} + z_c$$

$$= 0 + \frac{V^2}{2g} + 0$$

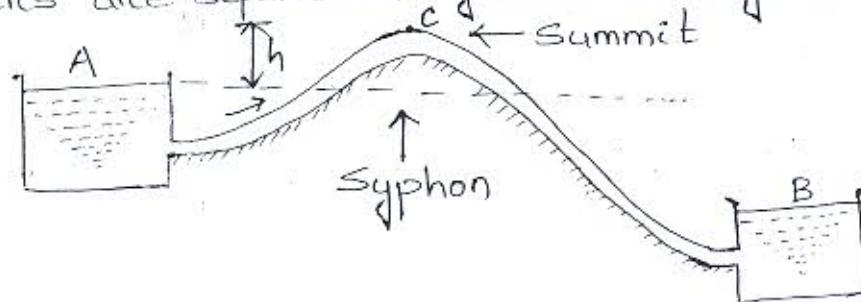
$$= \frac{2.734^2}{2 \times 9.81} = 0.38\text{m}$$

(b) Hydraulic gradient line will differ by $\frac{V^2}{2g}$ and drawn parallel to HGL.

7/09/20

Flow through pipe :- Syphon :-

Syphon is a long bent pipe which is used to transfer liquid from a reservoir at a higher elevation to another reservoir at a lower level when the two reservoirs are separated by a hill or high level ground.



The point C which is at the highest of the syphon is called the summit. As the point C is above the free surface of the water in the tank A, the pressure at C will be less than atmospheric pressure.

→ Theoretically, the pressure at C may be reduced to -10.3m of water but in actual practice this pressure is only -7.6 m of water.

$$\begin{aligned}\text{Then absolute pressure} &= 10.3 - 7.6 \\ &= 2.7 \text{ m.}\end{aligned}$$

→ If the pressure at C becomes less than 2.7 m of water absolute, the dissolved air and other gases would come out from water and collect at the summit.

→ The flow of water will be obstructed.

Syphon

Syphon is used for the following cases.

- (i) To carry water from one reservoir to another reservoir separated by a hill or ridge.
- (ii) To take out the liquid from a tank which is not having any outlet.
- (iii) To empty a channel not provided with any outlet sluice.

Q) A syphon of dia 200 mm connects two reservoirs having a difference in elevation of 20m. The length of the syphon is 500m and the summit is 3m above the water level on the upper reservoir. The length of the pipe from upper reservoir to the summit is 100m. Determine the discharge through the syphon and also pressure at the summit. Neglect minor losses. The co-eff. of friction, $f=0.005$.

Given, dia of syphon $\Rightarrow d = 200 \text{ mm} = 0.20 \text{ m}$

difference in level, $H = 20 \text{ m}$

Length of syphon, $L = 500 \text{ m}$

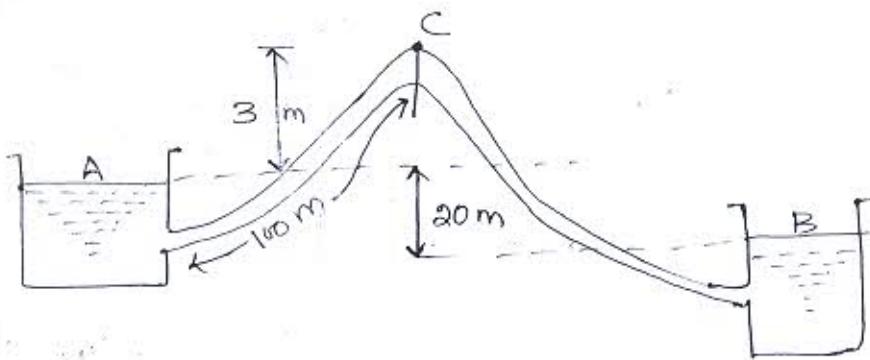
Ht. of summit from upper reservoir,
 $h = 3 \text{ m}$

Length of syphon upto summit

$$L_1 = 100 \text{ m}$$

$$f = 0.005$$

(b)



Applying Bernoulli's eqn to points A and B,

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + z_B + h_f$$

$$\Rightarrow 0 + 0 + z_A = 0 + 0 + z_B + h_f$$

$P_A = P_B$ = atmospheric pressure

$$V_A = V_B = 0$$

$$\Rightarrow z_A - z_B = h_f = \frac{4 f L V^2}{2 g d}$$

$$\Rightarrow 20m = \frac{4 \times 0.005 \times 500 \times V^2}{2 \times 9.81 \times 0.20}$$

$$= 2.548 V^2$$

$$\Rightarrow V = \sqrt{\frac{20}{2.548}} = 2.8 \text{ m/s}$$

Discharge, $Q = A \times V$

$$= \frac{\pi}{4} \times 0.2^2 \times 2.8 = 87.9 \text{ lit/s.}$$

Pressure at summit :-

Applying Bernoulli's eqn at A and C,

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{P_C}{\rho g} + \frac{V_C^2}{2g} + z_C + h_f$$

$$\Rightarrow 0 + 0 + 0 = \frac{P_C}{\rho g} + \frac{V^2}{2g} + 3 + h_f$$

$$\Rightarrow \frac{P_C}{\rho g} + \frac{2.8^2}{2 \times 9.81} + 3 + \frac{4 \times 0.005 \times 100 \times 2.8^2}{2 \times 9.81 \times 0.2} = 0$$

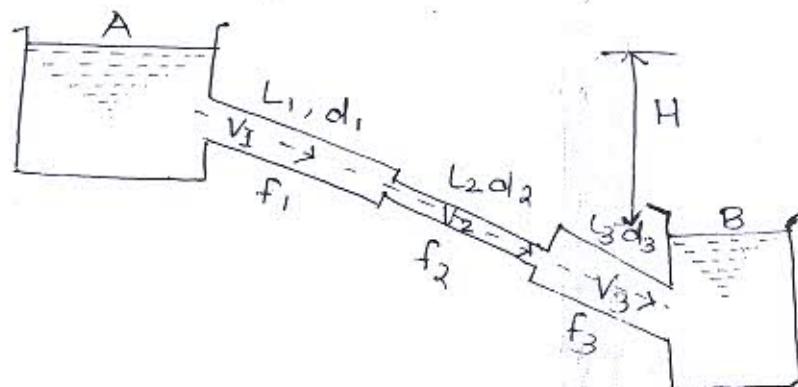
$$\Rightarrow \frac{P_c}{\rho g} + 7.399 = 0$$

$$\Rightarrow \frac{P_c}{\rho g} = -7.399 \text{ m of water},$$

9th Sept.

Flow through pipes in series or flow through Compound pipes

Pipes in series or compound pipes are defined as the pipes of different length and different diameters end to end to form a pipe line.



Let, L_1, L_2, L_3 = Length of pipe 1, 2 and 3 respectively

d_1, d_2, d_3 = dia of pipes 1, 2, 3 respectively.

v_1, v_2, v_3 = Velocity of flow through pipes 1, 2, 3

f_1, f_2, f_3 = co-eff. of friction for pipes 1, 2, 3

H = difference of water level.

The discharge passing through each pipe is same.

$$Q = A_1 V_1 = A_2 V_2 = A_3 V_3$$

The difference in liquid surface levels is equal to the sum of the total head loss in the pipes.

$$H = \frac{0.5 V_1^2}{2g} + \frac{4f_1 L_1 V_1^2}{2gd_1} + \frac{0.5 V_2^2}{2g} + \frac{4f_2 L_2 V_2^2}{2gd_2} \\ + \frac{(V_2 - V_3)^2}{2g} + \frac{4f_3 L_3 V_3^2}{2gd_3} + \frac{V_3^2}{2g}$$

If minor losses are neglected, then above equation becomes, as,

$$H = \frac{4f_1 L_1 V_1^2}{2gd_1} + \frac{4f_2 L_2 V_2^2}{2gd_2} + \frac{4f_3 L_3 V_3^2}{2gd_3}$$

If the coefficient of friction is same for all pipes,

$$f_1 = f_2 = f_3 = f,$$

Then the equation becomes,

$$H = \frac{4f L_1 V_1^2}{2gd_1} + \frac{4f L_2 V_2^2}{2gd_2} + \frac{4f L_3 V_3^2}{2gd_3}$$

$$\boxed{H = \frac{4f}{2g} \left[\frac{L_1 V_1^2}{d_1} + \frac{L_2 V_2^2}{d_2} + \frac{L_3 V_3^2}{d_3} \right]}$$

Q) The difference in water surface level in two tanks, which are connected by three pipes in series of length 300 m, 170 m and 210 m and of diameters 300 mm, 200 mm and 400 mm respectively is 12 m. Determine the rate of flow of water if co-efficient of friction are 0.005, 0.0052 and 0.0048 respectively. Considering (i) minor losses (ii) neglecting minor losses.

Given,

Difference of water level, $H = 12 \text{ m}$

Length of pipe, 1 $\Rightarrow L_1 = 300 \text{ m}$

$$d_1 = 300 \text{ mm} \\ = 0.3 \text{ m}$$

for pipe 2 $\Rightarrow L_2 = 170 \text{ m}$

$$d_2 = 200 \text{ mm} \\ = 0.2 \text{ m}$$

for pipe 3 $\Rightarrow L_3 = 210 \text{ m}$

$$d_3 = 400 \text{ mm} \\ = 0.4 \text{ m}$$

$$f_1 = 0.005 \quad f_2 = 0.0052 \quad f_3 = 0.0048$$

(i) Considering minor Losses :-

$$A_1 V_1 = A_2 V_2 = A_3 V_3$$

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{\frac{\pi}{4} \times d_1^2}{\frac{\pi}{4} \times d_2^2} \times V_1$$

$$= \frac{0.3^2}{0.2^2} \times V_1$$

$$= 2.25 V_1$$

$$V_3 = 0.5625 V_1$$

$$H = \frac{0.5 V_1^2}{2g} + \frac{4 f_1 L_1 V_1^2}{2g} + \frac{0.5 V_2^2}{2g} + \frac{4 f_2 L_2 V_2^2}{2g} + \frac{(V_2 - V_3)^2}{2g} + \frac{4 f_3 L_3 V_3^2}{2g} \times \frac{V_3^2}{2g}$$

$$\Rightarrow 12 = \frac{0.5 V_1^2}{2g} + \frac{4 \times 0.005 \times 300 \times V_1^2}{0.3 \times 2g} + \frac{0.6 \times (2.25 V_1^2)^2}{2g}$$

$$+ 4 \times 0.0052 \times 170 \times \frac{(2.25 V_1)^2}{2g \times 0.2} + \frac{(2.25 V_1 - 0.5625 V_1)^2}{2g}$$

$$+ \frac{4 \times 0.0048 \times 210 \times (0.5625 V_1)^2}{2g \times 0.4} + \frac{(0.5625 V_1)^2}{2g}$$

$$\Rightarrow 12 = \frac{V_1^2}{2g} [118.887]$$

$$\Rightarrow V_1 = \sqrt{\frac{12 \times 2 \times 9.81}{118.887}}$$

$$= 1.407 \text{ m/s.}$$

Discharge, $Q = \text{Area} \times \text{Velocity}$

$$= \frac{\pi}{4} \times 0.3^2 \times 1.407$$

$$= 0.09945 \text{ m}^3/\text{s.}$$

$$= 99.45 \text{ l/s}$$

(ii) Neglecting minor losses :-

$$H = \frac{4 f_1 L_1 V_1^2}{2 g d_1} + \frac{4 f_2 L_2 V_2^2}{2 g d_2} + \frac{4 f_3 L_3 V_3^2}{2 g d_3}$$

$$\Rightarrow 12 = \frac{V_1^2}{2g} \left[\frac{4 \times 0.005 \times 300}{0.3} + \frac{4 \times 0.0052 \times 170 \times 2.25}{0.2} \right. \\ \left. + \frac{4 \times 0.0048 \times 210 \times 0.5625}{0.4} \right]$$

$$\Rightarrow V_1 = \sqrt{\frac{2 \times 9.81 \times 12}{112.694}}$$

$$= 1.445 \text{ m/s}$$

Discharge $Q = V_1 \times A$

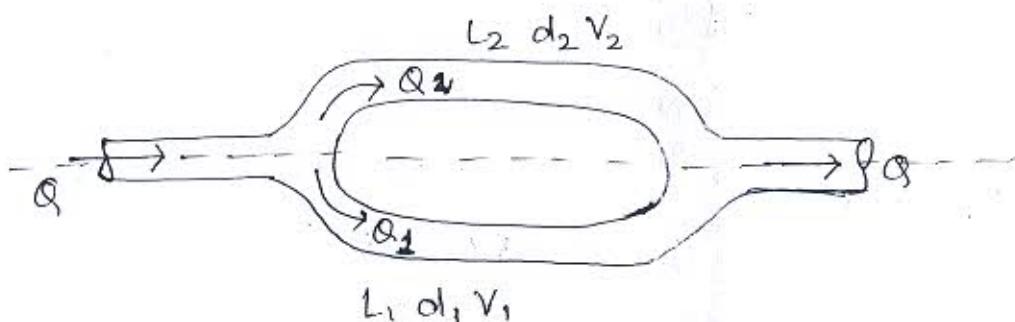
$$= 1.445 \times \frac{\pi}{4} \times 0.3^2$$

$$= 0.1021 \text{ m}^3/\text{s}$$

Flow through parallel pipes

Consider a main pipe which divides into two or more branches and again join together downstream to form a single pipe, then the branch pipe are said to be connected in parallel.

⇒ The discharge through the main is increased by connecting pipes in parallel.



$$\text{Total discharge} \Rightarrow Q = Q_1 + Q_2$$

Loss of head in each branch pipe is same.

$$H_1 = H_2$$

$$\Rightarrow \frac{4 f_1 L_1 V_1^2}{2 g d_1} = \frac{4 f_2 L_2 V_2^2}{2 g d_2}$$

$$f_1 = f_2$$

then,

$$\boxed{\frac{L_1 V_1^2}{2 g d_1} = \frac{L_2 V_2^2}{2 g d_2}}$$

Q) A main pipe is divided into two parallel pipes which again forms one pipe. The length and dia for the first parallel pipe are 2000 m and 1m respectively, while the length and dia of 2nd parallel pipe are 2000m and 0.8m. Find the rate of flow in each parallel pipe, if total flow in the main is 3 m³/s. The co-efficient of friction for each parallel pipe is same and equal to 0.005.

Given,

$$\text{Length of pipe } 1, L_1 = 2000 \text{ m}$$

$$\text{Dia of pipe } 1, d_1 = 1 \text{ m}$$

$$\text{Length of pipe } 2, L_2 = 2000 \text{ m}$$

$$d_2 = 0.8 \text{ m}$$

$$Q = 3 \text{ m}^3/\text{s}$$

$$f_1 = f_2 = f = 0.005$$

$$\text{Total discharge } \Rightarrow Q = Q_1 + Q_2 = 3$$

$$\frac{4f_1 L_1 V_1^2}{2g d_1} = \frac{4f_2 L_2 V_2^2}{2g d_2}$$

$$\Rightarrow \frac{4 \times 0.005 \times 2000 \times V_1^2}{2 \times 9.81 \times 1} = \frac{4 \times 0.005 \times 2000 \times V_2^2}{2 \times 9.81 \times 0.8}$$

$$\Rightarrow \frac{V_1^2}{1} = \frac{V_2^2}{0.8}$$

$$Q_1 = \frac{\pi}{4} \times d_1^2 \times V_1$$

$$= 1.906 \text{ m}^3/\text{s}$$

$$\Rightarrow V_1^2 = \frac{V_2^2}{0.8}$$

$$Q_2 = Q - Q_1$$

$$\Rightarrow V_1 = \frac{V_2}{0.894}$$

$$= 3 - 1.906$$

$$Q_1 = \frac{\pi}{4} \times d_1^2 \times V_1 = \frac{\pi}{4} \times 1^2 \times \frac{V_2}{0.894}$$

$$= 1.096 \text{ m}^3/\text{s}$$

$$Q_2 = \frac{\pi}{4} \times d_2^2 \times V_2 = \frac{\pi}{4} \times 0.64 \times V_2$$

$$Q = Q_1 + Q_2$$

$$\Rightarrow 3 = \frac{\pi}{4} \times \frac{V_2}{0.894} + \frac{\pi}{4} \times 0.64 \times V_2$$

$$\therefore V_2 = \frac{3}{\frac{\pi}{4} + 0.64} = 2.17 \text{ m/s}$$

Equivalent Pipe :-

12th Sept.

This is defined as the pipe of uniform diameter having loss of head and discharge equal to the loss of head and discharge of a compound pipe consisting of several pipes of different lengths and diameters.

→ The uniform diameter of the equivalent pipe is called equivalent size of pipe.

→ The length of equivalent pipe is equal to sum of lengths of the compound pipe consisting of different pipes.

Total head loss in the compound pipe, neglecting minor losses,

$$H = \frac{4f_1 L_1 V_1^2}{2g d_1} + \frac{4f_2 L_2 V_2^2}{2g d_2} + \frac{4f_3 L_3 V_3^2}{2g d_3}$$

Assuming,

$$f_1 = f_2 = f_3 = f$$

Discharge,

$$Q = A_1 V_1 = A_2 V_2 = A_3 V_3$$

$$= \frac{\pi}{4} \times d_1^2 V_1 = \frac{\pi}{4} \times d_2^2 V_2 = \frac{\pi}{4} \times d_3^2 V_3$$

$$\Rightarrow V_1 = \frac{4Q}{\pi d_1^2}, \quad V_2 = \frac{4Q}{\pi d_2^2}, \quad V_3 = \frac{4Q}{\pi d_3^2}$$

$$H = \text{head loss} = \frac{4fL V^2}{2gd}$$

$$H = \frac{4f L_1 \left(\frac{4Q}{\pi d_1^2}\right)^2}{2gd_1} + \frac{4f L_2 \left(\frac{4Q}{\pi d_2^2}\right)^2}{2gd_2} + \frac{4f L_3 \left(\frac{4Q}{\pi d_3^2}\right)^2}{2gd_3}$$

$$\Rightarrow = \frac{4 \times 16 Q^2 f}{2g \pi^2} \left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5} \right]$$

$$H = \frac{4fL \left(\frac{uQ}{\pi d^2} \right)^2}{2gd} = \text{head loss in equivalent pipe}$$

$$= \frac{4 \times 16 Q^2 f}{2g \pi^2} \left[\frac{L}{d^5} \right]$$

By comparing these two values,

We get,

$$\frac{4 \times 16 Q^2 f}{2g \pi^2} \left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5} \right] = \frac{4 \times 16 Q^2 f}{\pi^2 2g} \left[\frac{L}{d^5} \right]$$

$$\Rightarrow \boxed{\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5} = \frac{L}{d^5}}$$

The above equation is known as Dupuit's eqn

Q) Three pipes of lengths 800m, 500m and 400m and of diameters 500mm, 400mm and 300mm respectively are connected in series. These pipes are to be replaced by a single pipe of lengths 1700m. Find the dia of the single pipe.

$$\Rightarrow \text{Given, } L_1 = 800 \text{ m} \quad d_1 = 500 \text{ mm} = 0.5 \text{ m}$$

$$L_2 = 500 \text{ m} \quad d_2 = 400 \text{ mm} = 0.4 \text{ m}$$

$$L_3 = 400 \text{ m} \quad d_3 = 300 \text{ mm} = 0.3 \text{ m}$$

$$\text{Length of single pipe} = L_1 + L_2 + L_3 = 1700 \text{ m.}$$

$$\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$$

$$\Rightarrow \frac{1700}{d^5} = \frac{800}{0.5^5} + \frac{500}{0.4^5} + \frac{400}{0.3^5}$$

$$\Rightarrow d^5 = \frac{1700}{239037}$$

$$= 0.007118$$

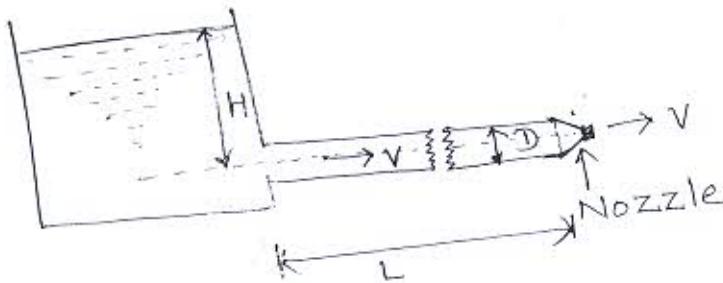
$$\Rightarrow d = 0.3718$$

$$= 371.8 \text{ mm.}$$

14th Sept

Flow through Nozzles

- A nozzle is fitted at the end of the pipe.
- The total energy at the end of the pipe consists of pressure energy and kinetic energy.
- By fitting the nozzle at the end of the pipe, the total energy is converted into kinetic energy.
- Thus nozzles are used, where higher velocity of flow are required.



- The examples are,
- (i) In case of pelton turbine, the nozzle is fitted at the end of the pipe (called penstock), to increase velocity.
- (ii) In case of the extinguishing fire, a nozzle is fitted at the end of the hose pipe to increase velocity.

Let, D = diameter of the pipe

L = length of the pipe

$$A = \text{Area of the pipe} = \frac{\pi}{4} D^2$$

V = Velocity of flow in pipe

H = total head at the inlet of the pipe

d = dia of nozzle at outlet

v = velocity of flow at outlet of nozzle

a = area of the nozzle at outlet

$$= \frac{\pi}{4} d^2$$

$$\text{Head loss due to friction, } h_f = \frac{4f L v^2}{2g D}$$

Head loss

Power transmitted through Nozzle :-

The KE of the jet at the outlet of nozzle

$$= \frac{1}{2}mv^2$$

Now mass of liquid at the outlet of nozzle
per sec = ρav

∴ KE of the jet at the outlet per sec.
 $= \frac{1}{2} \rho av \times v^2 = \frac{1}{2} \rho av^3$

power in KW at the outlet of nozzle

$$\eta = \frac{\text{Power at outlet of nozzle}}{\text{Power at inlet of pipe}}$$

$$= \frac{\frac{1}{2} \rho av^3}{1000}$$

$$\frac{\rho g Q H}{1000}$$

$$= \frac{\frac{1}{2} \rho av^3}{\rho g \cdot Q \cdot H}$$

$$= \frac{v^2}{2gH}$$

$$= \frac{1}{1 + \frac{4fL}{D} \times \frac{A_2^2}{A_1^2}}$$

Condition for maximum power transmitted

through nozzle

It states that power transmitted through nozzle is maximum when the head lost due to friction in pipe is one-third the total total head supplied at the inlet of pipe.

$$h_f = H/3$$

Diameter of nozzle for maximum transmission of power through nozzle

$$\frac{A}{a} = \sqrt{\frac{8fL}{D}}$$

$$d^4 = \frac{D^5}{8fL}$$

- Q) A nozzle is fitted at the end of the pipe of length 300 m and of diameter 100 mm. For the maximum transmission of power through the nozzle, find the dia of nozzle. Take $f = 0.009$.

Given, $L = 300\text{m}$

$$D = 100\text{mm} = 0.1\text{m}$$

$$f = 0.009$$

For maximum transmission of power, the dia of nozzle is given by relation,

$$\begin{aligned} d &= \left(\frac{D^5}{8fL} \right)^{1/4} \\ &= \left(\frac{0.1^5}{8 \times 0.009 \times 300} \right)^{1/4} \\ &= 26.08\text{ mm.} \end{aligned}$$